Exercise 57

For the following exercises, sketch a graph of the quadratic function and give the vertex, axis of symmetry, and intercepts.

$$f(x) = -2x^2 + 5x - 8$$

Solution

In order to more easily graph the quadratic function, write it in vertex form by completing the square. The following algebraic identity is necessary.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Factor the coefficient of x^2.

$$f(x) = -2\left(x^2 - \frac{5}{2}x + 4\right)$$

Notice that $2B = -\frac{5}{2}$, which means $B = -\frac{5}{4}$ and $B^2 = \frac{25}{16}$. Add and subtract $\frac{25}{16}$ within the parentheses and use the identity.

$$\begin{aligned} f(x) &= -2\left[\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) + 4 - \frac{25}{16}\right] \\ &= -2\left[\left(x + \left(-\frac{5}{4}\right)\right)^2 + \frac{39}{16}\right] \\ &= -2\left(x - \frac{5}{4}\right)^2 - \frac{39}{8} \end{aligned}$$

Therefore, the vertex is $\left(\frac{5}{4}, -\frac{39}{8}\right)$, and the axis of symmetry is $x = \frac{5}{4}$. To determine the y-intercept, set $x = 0$.

$$f(0) = -2\left(0 - \frac{5}{4}\right)^2 - \frac{39}{8} = -2\left(\frac{25}{16}\right) - \frac{39}{8} = -\frac{64}{8} = -8$$

Therefore, the y-intercept is $(0, -8)$. To get the x-intercept, set $y = 0$ and solve the equation for x.

$$0 = -2\left(x - \frac{5}{4}\right)^2 - \frac{39}{8}$$

$$2\left(x - \frac{5}{4}\right)^2 = -\frac{39}{8}$$

$$\left(x - \frac{5}{4}\right)^2 = -\frac{39}{16}$$

Since the square is equal to a negative number, there are no real solutions for x, which means there are no x-intercepts.

A graph of the function is shown below.